Grade Level/Course: Algebra 1

Lesson/Unit Plan Name: Graphing Piecewise Functions

Rationale/Lesson Abstract: Students will graph piecewise defined functions using three different methods.

Timeframe: 1 to 2 Days (60 minute periods)

Common Core Standard(s): F-IF.7b Graph square root, cube root, and piece-wise defined functions, including step functions and absolute value functions.

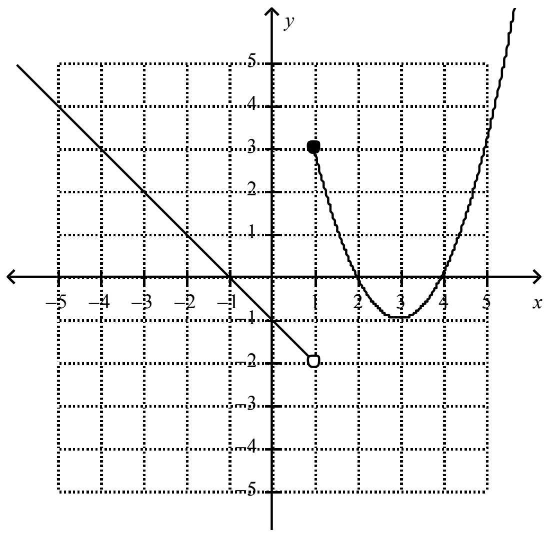
Notes: The Warm-Up is on page 8.

A graphing handout is provided specifically for example 1 (method 1) on page 6.

Instructional Resources/Materials: Graph paper or coordinate plane handout, rulers.

Lesson:

Think-Pair-Share: Describe the graph below. Then compare it to other graphs we have seen in this class.



Possible Descriptions:

- The graph is a function.
- The graph is composed of part of a line and a part of a parabola.
- The graph is not continuous, there is a break in the graph at $x = 1$.

Further Discussion:
Show the equation of the graph and discuss how it relates to the graph.

$$f(x) = \begin{cases} -x - 1 & ,\text{if } \ x < 1 \\ x^2 - 6x + 8 & ,\text{if } \ x \geq 1 \end{cases}$$

*Notice the structure of the function, after each function you see a restricted domain.

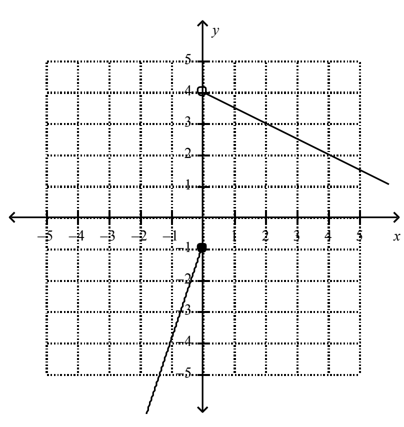
A <u>piecewise function</u> is a function represented by two or more functions, each corresponding to a part of the domain.

A piecewise function is called piecewise because it acts differently on different "pieces" of the number line.

<u>Example 1:</u> Graph the piecewise function $f(x) = \begin{cases} 3x - 1 & ,\text{if } \ x \leq 0 \\ -\frac{1}{2}x + 4 & ,\text{if } \ x > 0 \end{cases}$.

Think-Pair: Predict what the graph will look like.

Solution:



<u>Method 1 (Uses 3 coordinate planes- see p. 6):</u>
(Complete use of method 1 shown on page 7)

- Identify the two functions that create the piecewise function.

$$y = 3x - 1$$

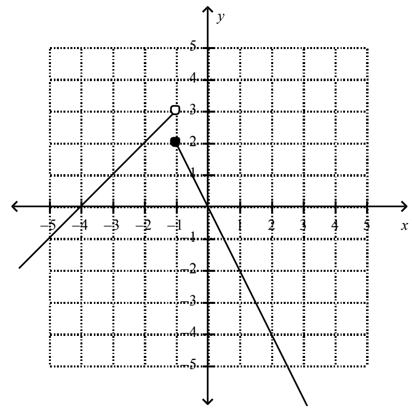
$$y = \frac{1}{2}x + 4$$

- Graph each function separately.
- Identify the break between each function as given by the domain of the piecewise function.
- Use a different color to highlight the piece of the graph that is given by the domain of the piecewise function.
- On the third graph, graph the piecewise function. To the left of $x = 0$ and including $x = 0$, graph $y = 3x - 1$. To the right of $x = 0$ and excluding $x = 0$, graph $y = \frac{1}{2}x + 4$.

Try: Graph the function $f(x)=\begin{cases} x+4 & ,\text{if } x<-1 \\ -2x & ,\text{if } x\geq -1 \end{cases}$.

Think-Pair: What kind of function is this? Explain.
Predict what the graph will look like.

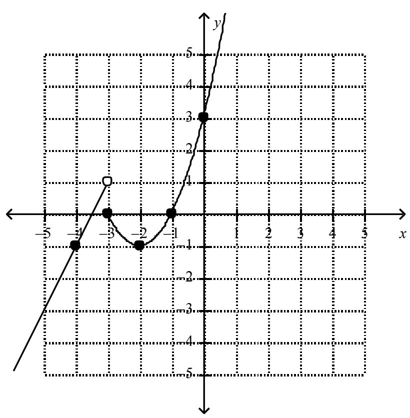
Solution:



Example 2: Graph the function $f(x)=\begin{cases} 2x+7 & ,\text{if } x<-3 \\ x^2+4x+3 & ,\text{if } x\geq -3 \end{cases}$.

Think-Pair: Predict what the graph will look like.

Solution:



Method 2 (Uses 1 coordinate plane):

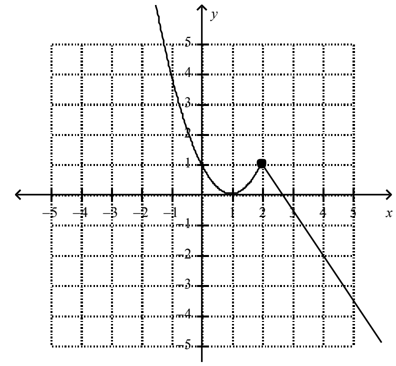
- Split the domain of the piecewise function into three sections.
- Identify the function corresponding to each section.
- Find points on $f(x)$ by substituting values of the domain in each piece.
- For $x=-3$, find the output for both equations*. Identify each point as open or closed.
- Graph the points.

$x<-3$	$x=-3$	$x>-3$
$f(x)=2x+7$ $f(-4)=2(-4)+7$ $=-8+7$ $=-1$ $(-4,-1)$ $f(-5)=2(-5)+7$ $=-10+7$ $=-3$ $(-5,-3)$	$f(x)=x^2+4x+3$ $f(x)=(x+3)(x+1)$ $f(-3)=(-3+3)(-3+1)$ $=0$ $(-3,0)$ $(-3,0)$ is a closed point on $f(x)$. $y=2x+7$ $*\ y=2(-3)+7$ $y=-6+7$ $y=1$ $(-3,1)$ is an open point on $f(x)$.	$f(x)=x^2+4x+3$ $f(x)=(x+3)(x+1)$ $f(-2)=(-2+3)(-2+1)$ $=(1)(-1)$ $=-1$ $(-2,-1)$ $f(-1)=(-1+3)(-1+1)$ $=(2)(0)$ $=0$ $(-1,0)$ $f(0)=(0+3)(0+1)$ $=(3)(1)$ $=3$ $(0,3)$

***Discuss:** We input $x=-3$ into $y=2x+7$ even though $f(x)\neq 2x+7$ at $x=-3$ because the open circle will occur at $x=-3$.

Try: Graph the function $f(x)=\begin{cases}(x-1)^2 & ,\text{if } x\le 2\\ -\frac{3}{2}x+4 & ,\text{if } x>2\end{cases}$.

Solution:



Think-Pair: What kind of function is this? Explain.
Predict what the graph will look like.
Which method did you use?
How is this graph different from the previous graphs?

Example 3: The function below describes the price of a movie ticket (in dollars) depending on the age of the person (in years). Graph $p(x)$.

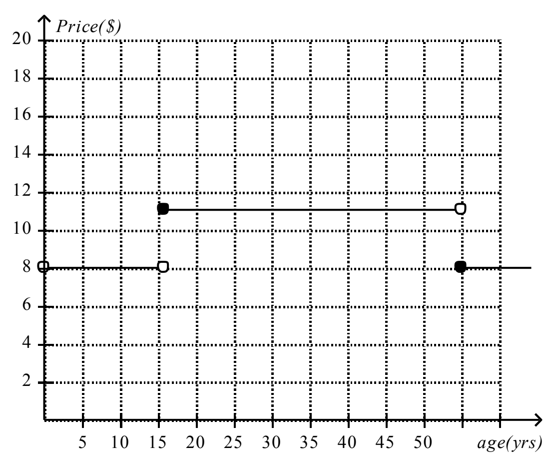
$$p(x)=\begin{cases}8 & ,\text{if } 0<x<16\\ 11 & ,\text{if } 16\le x<55\\ 8 & ,\text{if } x\ge 55\end{cases}$$

Discuss the meaning of the function:

People under 16 years of age pay $8 per ticket
People who are at least 16 year of age, but younger than 55 years old pay $11 per ticket.
People who are 55 years old or older pay $8 per ticket.
What kind of functions are $y=8$ and $y=11$?

Think-Pair-Share: Predict what the graph is going to look like.

- The graph is going to be in the first quadrant.
- The graph will consist of three linear functions, which are all pieces of horizontal lines.
- The horizontal axis is labeled age.
- The vertical axis is labeled price.
- The axes will not be labeled by one's.



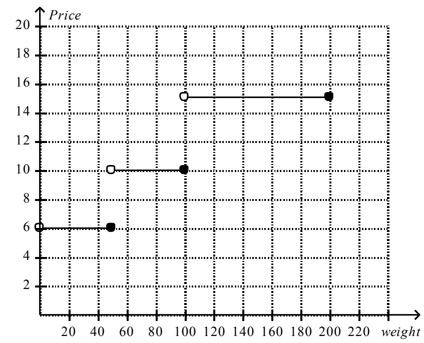
Method 3 (Direct Approach):

- Graph the horizontal line $y=8$ from $x=8$ to $x=16$. The point $(16, 8)$ is open.
- Graph the horizontal line $y=11$ from $x=16$ to $x=55$. The point $(16, 11)$ is closed and $(55, 11)$ is open.
- Graph the horizontal line $y=8$ from $x=55$ to infinity. The point $(55, 8)$ is closed.

Try: Graph the function $f(x)=\begin{cases}6 & ,\text{if } 0<x\le 50\\ 10 & ,\text{if } 50<x\le 100\\ 15 & ,\text{if } 100<x\le 200\end{cases}$.

Write a scenario represented by this function.

Solution:

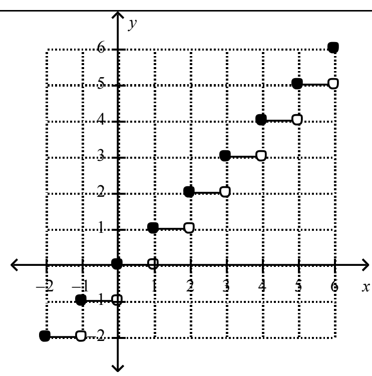
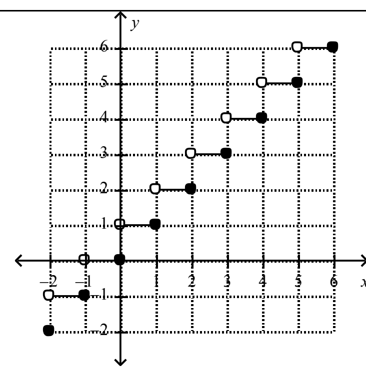


Possible scenario:
The function describes the cost to ship packages given the weight of the package. It cost \$6 to ship packages weighing 50 pounds or less, \$10 to ship packages weighing over 50 pounds up to 100 pounds, and \$15 to ship packages weighing over 100 pounds up to 200 pounds.

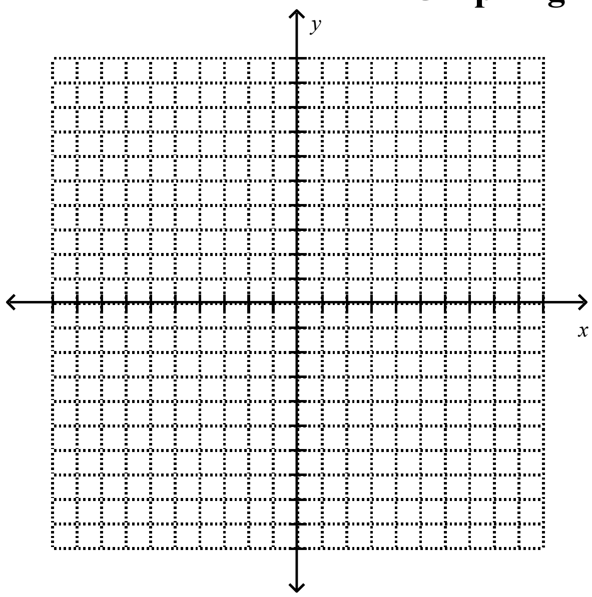
Think-Pair-Share: The functions in example 3 and Try 3 are a specific type of piecewise function called a step function. Why do you think they are called step functions?

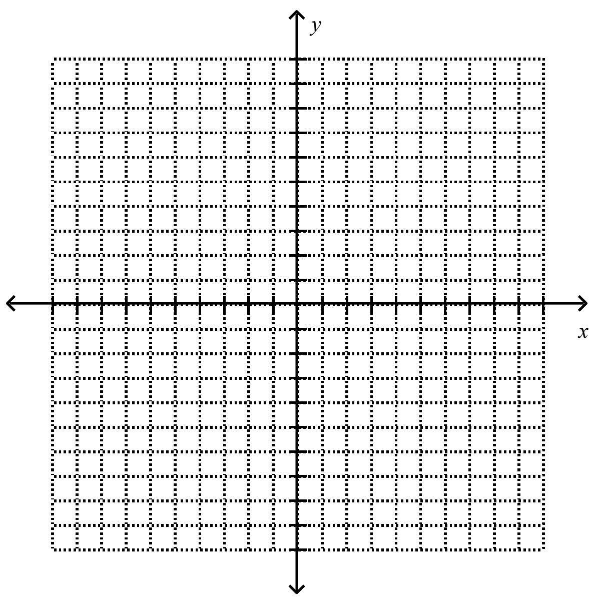
A <u>step function</u> is a piecewise function whose graph resembles a staircase or steps.

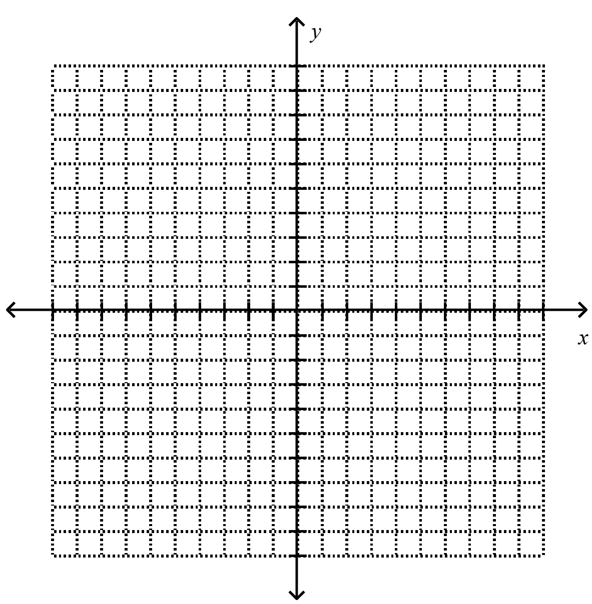
SPECIAL STEP FUNCTIONS:

The Greatest Integer Function, or The Floor Function $f(x)=\lfloor x \rfloor$	The Ceiling Function $f(x)=\lceil x \rceil$
Describes the largest integer not greater than *x*, or the largest integer less than or equal to *x*.	Describes the smallest integer not less than *x*.
	
An example of a floor function is a person's age. If someone is 15 years and 4 months old, the person would simply say that they are 15 years old.	An example of a ceiling function is a cell phone service. Suppose the company charges by the number of minutes. If you are on the phone for 2.7 minutes, the company will charge for 3 minutes.

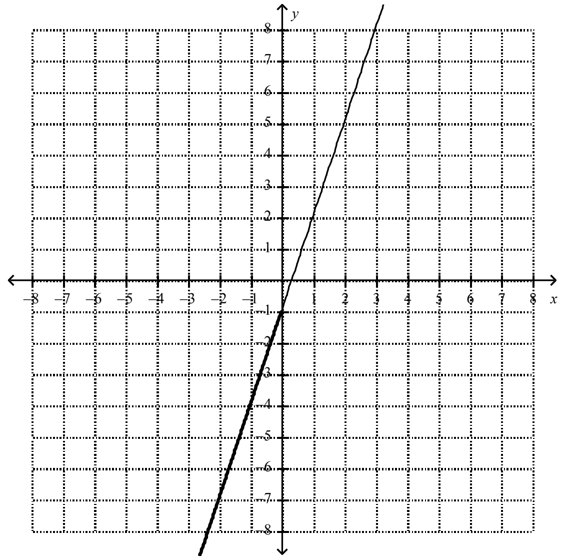
Graphing Piecewise Functions

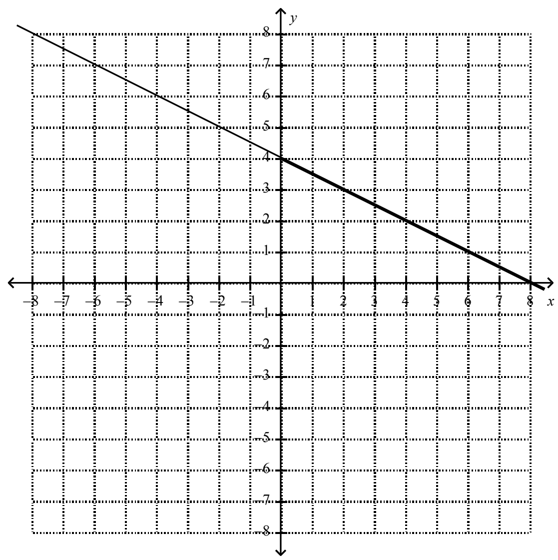


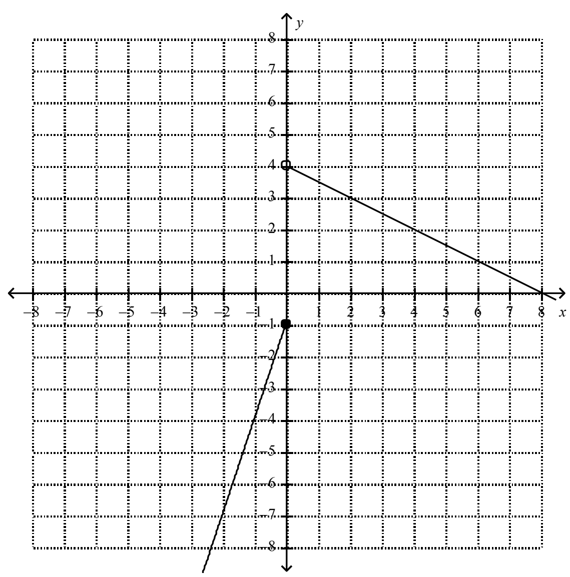




Graphing Piecewise Functions – Ex. 1 Sample







Warm-Up

Review: CA Alg. 1 CCSS F-IF.2

Given the function $f(x) = x^2 - 2x + 5$, find the following function values:

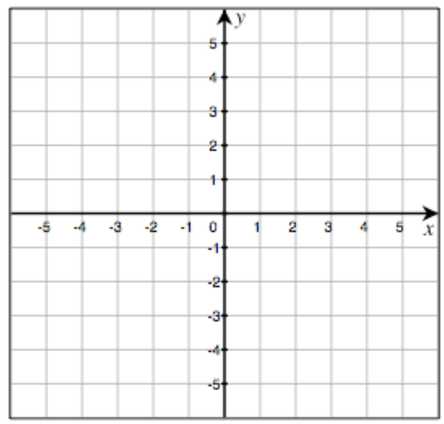
a) $f(0)$

b) $f(-3)$

c) $f(4)$

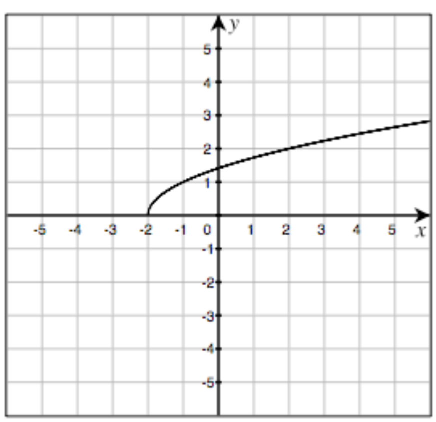
Current: CA Alg. 1 CCSS F-IF.7a

Graph the function $f(x) = x^2$ for $x \geq 0$.



Current: CA Alg. 1 CCSS F-IF.1

Find the domain of the function shown in the graph below.



Domain:

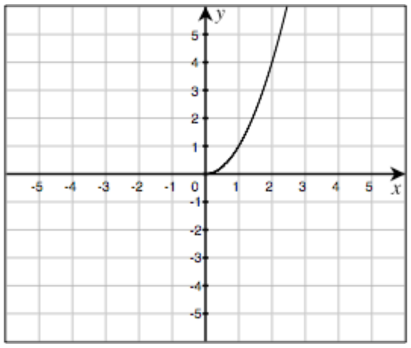
Current: CA Alg. 1 CCSS C-CED.2

For the function graphed to the left in quadrant III, write the rule for the function.

$f(x) =$

Solutions to Warm-Up:

Quadrant I



Quadrant II

Given the function $f(x) = x^2 - 2x + 5$,
find the following function values:

a)

$$f(0) = (0)^2 - 2(0) + 5$$
$$f(0) = 5$$

b)

$$f(-3) = (-3)^2 - 2(-3) + 5$$
$$f(-3) = 9 + 6 + 5$$
$$f(-3) = 15 + 5$$
$$f(-3) = 20$$

c)

$$f(4) = (4)^2 - 2(4) + 5$$
$$f(4) = 16 - 8 + 5$$
$$f(4) = 8 + 5$$
$$f(4) = 13$$

Quadrant III

Domain: $x \geq -2$

Quadrant IV

$f(x) = \sqrt{x+2}$